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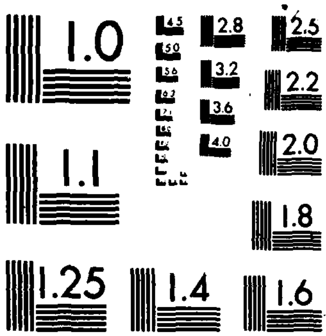

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SIMPLIFIED NAVIER-STOKES EQUATIONS AND COMBINED SOLUTION OF NON-VISCOUS AND VISCOUS BOUNDRY LAYER EQUATIONS

by

Gao Zhi

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Block	Italic	Transliteration	Block	Italic	Transliteration
А а	*А а*	A, a	Р р	*Р р*	R, r
Б б	*Б б*	B, b	С с	*С с*	S, s
В в	*В в*	V, v	Т т	*Т т*	T, t
Г г	*Г г*	G, g	У у	*У у*	U, u
Д д	*Д д*	D, d	Ф ф	*Ф ф*	F, f
Е е	*Е е*	Ye, ye; E, e*	Х х	*Х х*	Kh, kh
Ж ж	*Ж ж*	Zh, zh	Ц ц	*Ц ц*	Ts, ts
З з	*З з*	Z, z	Ч ч	*Ч ч*	Ch, ch
И и	*И и*	I, i	Ш ш	*Ш ш*	Sh, sh
Й й	*Й й*	Y, y	Щ щ	*Щ щ*	Shch, shch
К к	*К к*	K, k	Ъ ъ	*Ъ ъ*	"
Л л	*Л л*	L, l	Ы ы	*Ы ы*	Y, y
М м	*М м*	M, m	Ь ь	*Ь ь*	'
Н н	*Н н*	N, n	Э э	*Э э*	E, e
О о	*О о*	O, o	Ю ю	*Ю ю*	Yu, yu
П п	*П п*	P, p	Я я	*Я я*	Ya, ya

*ye initially, after vowels, and after ъ, ь; e elsewhere.
When written as ё in Russian, transliterate as yё or ё.

RUSSIAN AND ENGLISH TRIGONOMETRIC FUNCTIONS

Russian	English	Russian	English	Russian	English
sin	sin	sh	sinh	arc sh	$\sinh^{-1}$
cos	cos	ch	cosh	arc ch	$\cosh^{-1}$
tg	tan	th	tanh	arc th	$\tanh^{-1}$
ctg	cot	cth	coth	arc cth	$\coth^{-1}$
sec	sec	sch	sech	arc sch	sech^{-1}
cosec	csc	csch	csch	arc csch	csch^{-1}

Russian	English
rot	curl
lg	log

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SIMPLIFIED NAVIER-STOKES EQUATIONS AND COMBINED SOLUTION OF NON-VISCOUS AND VISCOUS BOUNDARY LAYER EQUATIONS

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Abstract

This paper presents a part of the technical report[2] in which the author studied simplified Navier-Stokes equations and the combined solution of non-viscous and boundary layer equations. From the full Navier-Stokes equations and an analysis of the combined solution of non-viscous and boundary layer equations, simplified Navier-Stokes equations were worked out. A perturbation analysis which differs slightly from the match-perturbation-expansions of inner-outer layers developed by Van Dyke[1] shows that the solution of the simplified Navier-Stokes equations in uniformly valid with accuracy of $O(Re_\infty{}^{-1/2})$ in the whole flow field, where $Re_\infty = \frac{\rho_\infty U_\infty L}{\mu_\infty}$, is the density of free stream, U_∞ the x-component of velocity, L the characteristic length, μ_∞ the dynamic viscosity of free steam.

The simplified N-S equations possess the characteristics of parabolic-hyperbolic equations. We can use the forward advancing calculation method for stationary situations and it must be much simpler than numerically solving the elliptical complete N-S equations; by solving the simplified N-S equations, we can simultaneously calculate the non-viscous outer flow and viscous boundary layer flow. Theoretically, they must be superior to the conventional method of first calculating the non-viscous flow and later calculating the viscous boundary layer flow. The simplified N-S equations can truly reflect the mechanical appearances of many typical flow fields. For example, they can accurately calculate the complex flow chart etc. of the mutual perturbation between the viscous boundary layer, upper entropic layer and non-viscous outer flow in the hypersonic flow field around a body.

Developments in the study of solving the simplified N-S equations are still being made. In reality, the problems of its mathematical character, steadiness, accurate formulation of the Cauchy problem, accuracy which can be reached in calculating the flow field etc. have still not been perfectly resolved and the views of different authors are not in agreement. This paper presents part of the research contents done by the uathor in 1967 investigating the simplified N-S equations, brings forth simplified N-S equations starting from analysis of the combined solution of the non-viscous and viscous boundary layers and uses the perturbation method to prove that we can obtain a uniform solution with accuracy of $O(Re_{\infty}^{-1/2})$ quantitative level from these equations. It should be pointed out in passing that analagous analysis for deducing the simplified N-S equations were later seen in references such as Reference [3].

1. Simplified Navier-Stokes Equations

If x and y are separately the orthogonal coordinates along the wall surface and vertical wall surface (Fig. 1), u and v are the corresponding speed components, and ρ , p, T, μ and λ separately indicate the density, pressure, temperature, viscosity coefficient and thermal conductivity. The overtaking flow parameters U_{∞} and $\rho_{\infty} U_{\infty}^2$ are separately the characteristic quantities of the speed and pressure and fixed wall length L is the length's characteristic quantity; we assume the quantitative levels of ρ and μ can use ρ_{∞} and μ_{∞} for balance.

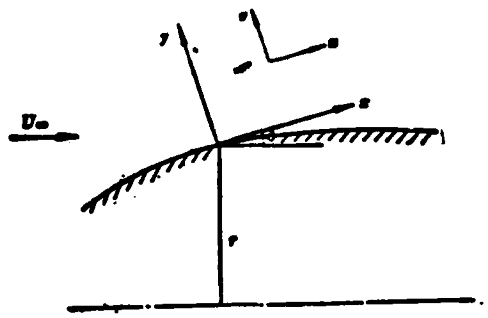

Fig. 1

Based on the two views of the non-viscous outer flow and viscous boundary layer flow, we carry out the following quantitative estimates of each term in the entire N-S equations:

$$
\begin{array}{lccccccccc}
 & \frac{u}{H}\frac{\partial u}{\partial x} + & v\frac{\partial u}{\partial y} + & \frac{kuv}{H} = & \frac{-1}{\rho H}\frac{\partial p}{\partial x} + & \frac{1}{\rho}\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right) + & \frac{\pi\mu}{\rho}\frac{\partial u}{\partial y} - & \frac{k\mu}{\rho}\frac{\partial}{\partial y}\left(\frac{\mu}{H}\right) + & \Phi_t \\
\text{(1) 无粘流} & \frac{U_\infty^2}{L} & \frac{U_\infty^2}{L} & \frac{U_\infty^2}{L} & \frac{U_\infty^2}{L} & Re_\infty^{-1}\frac{U_\infty^2}{L} & Re_\infty^{-1}\frac{U_\infty^2}{L} & Re_\infty^{-1}\frac{U_\infty^2}{L} & Re_\infty^{-1}\frac{U_\infty^2}{L} \\
\text{(2) 边界层流} & \frac{U_\infty^2}{L} & \frac{U_\infty^2}{L} & \frac{\delta U_\infty^2}{L^2} & \frac{U_\infty^2}{L} & Re_\infty^{-1}\frac{LU_\infty^2}{\delta^2} & Re_\infty^{-1}\frac{U_\infty}{\delta} & Re_\infty^{-1}\frac{U_\infty}{\delta} \leqslant & Re_\infty^{-1}\frac{U_\infty^2}{L}
\end{array}
\tag{1.1}
$$

$$
\begin{array}{lccccccccc}
 & \frac{u}{H}\frac{\partial v}{\partial x} + & v\frac{\partial v}{\partial y} - & \frac{ku^2}{H} = & -\frac{1}{\rho}\frac{\partial p}{\partial y} + & \frac{4}{3\rho}\frac{\partial}{\partial y}\left(\mu\frac{\partial v}{\partial y}\right) + & \frac{1}{\rho H}\frac{\partial}{\partial x}\left(\mu\frac{\partial u}{\partial y}\right) + & j\frac{\sin\theta}{\rho r H}\mu\frac{\partial u}{\partial y} + & \Phi_n \\
\text{(3) 无粘流} & \frac{U_\infty^2}{L} & \frac{U_\infty^2}{L} & \frac{U_\infty^2}{L} & \frac{U_\infty^2}{L} & Re_\infty^{-1}\frac{U_\infty^2}{L} & Re_\infty^{-1}\frac{U_\infty^2}{L} & Re_\infty^{-1}\frac{U_\infty^2}{L} & Re_\infty^{-1}\frac{U_\infty^2}{L} \\
\text{(4) 边界层流} & \frac{\delta U_\infty^2}{L^2} & \frac{\delta U_\infty^2}{L^2} & \frac{U_\infty^2}{L} & & Re_\infty^{-1}\frac{U_\infty^2}{\delta} & Re_\infty^{-1}\frac{U_\infty^2}{\delta} & Re_\infty^{-1}\frac{U_\infty^2}{\delta} \leqslant & Re_\infty^{-1}\frac{U_\infty^2}{L}
\end{array}
\tag{1.2}
$$

Key: (1) Non-viscous flow; (2) Boundary layer flow; (3) Non-viscous flow; (4) Boundary layer flow.

Here, $\pi = j\frac{\cos\theta}{r} + \frac{k}{H}$, j=0 (two dimensional flow, 1 (axially symmetric flow), H=1+ky, k is the wall surface curvature, $Re_\infty = \frac{\rho_\infty U_\infty L}{\mu_\infty}$, $\frac{\delta}{L} = O(Re_\infty^{-1/2})$, δ is the boundary layer thickness, Φ_t and Φ_n are separately the quantitative levels in the tangential and normal equations which are equal to or

smaller than the terms of the $O\left(Re_\infty^{-1}\frac{U_\infty^2}{L}\right)$ quantita tive level. We carry out the following discussion when the large Reynold's number $Re_\infty \gg 1$:

1) When $y > \delta$, we omit the terms with quantitative levels of of $O(Re_\infty \frac{U_\infty^2}{L})$ in the entire N-S equations and obtain the Euler equations; when $y \leqslant \delta$, we omit the terms with quantitative levels smaller than and equal to $O(Re_\infty^{-1/2} \frac{U_\infty^2}{L})$ and obtain the Prandtl boundary layer equations.

2) When jointly solving the Euler and boundary layer equations and there exists mathematical singularity on the boundary line of $y=\delta$, the asymptotic value of the solution of the Euler equations when $y(>\delta) \to \delta$ and the asymptotic value of the solution of the boundary layer equations when $y(<\delta) \to \delta$ cannot be completely the same and the singular real quantitative level is $O(Re_\infty^{-1/2} \frac{U_\infty^2}{L})$. It should be noted that when $y(>\delta) \to \delta$, the real quantitative level of the terms of inertia in the normal Euler equations is also $O(Re_\infty^{-1/2} \frac{U_\infty^2}{L})$.

3) Therefore, in order to eliminate the singularity which appears when jointly solving the non-viscous outer flow and viscous boundary layer flow, we should select and omit terms in the N-S equations which are equal to and smaller than $O(Re_\infty^{-1} \frac{U_\infty^2}{L})$ and afterwards obtain the equations. These are then the simplified N-S equations; by combining the solution of the simplified N-S equations and Euler equations we can eliminate the mathematical singularity and the obtained accuracy is the $O(Re_\infty^{-1} \frac{U_\infty^2}{L})$ quantitative level which is suitable for the uniform solution of the entire flow field.

As mentioned above, the simplified N-S equations point to the set of equations obtained after omitting the terms with quantitative levels equal to and smaller than $O(Re_\infty^{-1} \frac{U_\infty^2}{L})$ in the entire N-S equations estimated according to the boundary

layer viewpoint. We concretely write them (continuity and energy equations) as:

$$\frac{\partial}{\partial x}(\rho u r^j) + \frac{\partial}{\partial y}(\rho v H r^j) = 0 \qquad (1.3)$$

$$\rho\left(\frac{u}{H}\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + \frac{kuv}{H}\right) = -\frac{1}{H}\frac{\partial p}{\partial x} + \frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right)$$
$$+ \kappa\mu\frac{\partial u}{\partial y} - ku\frac{\partial}{\partial y}\left(\frac{\mu}{H}\right) \qquad (1.4)$$

$$\rho\left(\frac{u}{H}\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} - \frac{ku^2}{H}\right) = -\frac{\partial p}{\partial y} + \frac{4}{3}\frac{\partial}{\partial y}\left(\mu\frac{\partial v}{\partial y}\right)$$

$$+ \frac{1}{H}\frac{\partial}{\partial x}\left(\mu\frac{\partial u}{\partial y}\right) + j\frac{\mu\sin\theta}{rH}\frac{\partial u}{\partial y} \qquad (1.5)$$

$$\rho c_p\left(\frac{u}{H}\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) - \left(\frac{u}{H}\frac{\partial p}{\partial x} + v\frac{\partial p}{\partial y}\right)$$
$$= \frac{\partial}{\partial y}\left(\lambda\frac{\partial T}{\partial y}\right) + \kappa\left(\lambda\frac{\partial T}{\partial y} + \mu u\frac{\partial T}{\partial y}\right) + \mu\Phi_c \qquad (1.6)$$

In the formulas

$$\Phi_c = \left(\frac{\partial u}{\partial y}\right)^2 - \kappa u\frac{\partial u}{\partial y} - \frac{2ku}{H}\frac{\partial u}{\partial y} + ku^2\frac{\partial}{\partial y}\left(\frac{\mu}{H}\right)$$

c_p is the constant pressure specific heat. Simplified N-S equations (1.3)-(1.6) do not include the $\frac{\partial^2}{\partial x^2}$ term and when $\mu \neq 0$, it has four repeated characters and two non-zero real characters. Therefore, it has parabolic-hyperbolic equation characteristics [4] and for constant situations we can use the forward advancing calculation method. At the same time, we should note that when $y(<\delta) \to \delta$, $\frac{\partial}{\partial y}$ goes from $O(Re_\infty^{1/2}) \to O(1)$ and the quantitative levels of the viscous terms all change into $O(Re_\infty^{-1}\ \frac{U_\infty^2}{L})$; accuracy reaches the $O(Re_\infty^{-1/2}\ \frac{U_\infty^2}{L})$ quantitative level and when $y \geqslant \delta$, the simplified N-S equations will smoothly transit to be Euler equations. This is to say that in order to obtain complete flow field solutions with the accuracy of the $O(Re_\infty^{-1/2}\ \frac{U_\infty^2}{L})$ quantitative

level, we need not simultaneously solve the simplified N-A and Euler equations. In principle, it is only necessary to give the initial boundary value, which is the overtaking flow (or shock wave) and wall surface conditions, and independently solve the simplified N-S equations.

2. Discussion of Accuracy

When using quasi-linear simplified N-S equations to calculate the entire flow field including the non-viscous outer flow and viscous boundary layer, we can obtain a uniform solution with accuracy of $O(Re_{\infty}^{-1/2} \frac{U_{\infty}^2}{L})$. In order to prove this conclusion, it is necessary to use slightly different perturbation expansion from common inner and outer lay matched expansion [1]. The solution of entire N-S equations (1.1) and (1.2), when in the $0 \leqslant y \leqslant \delta$ viscous boundary layer, can expand into the following power series of $\varepsilon = Re_{\infty}^{-1/2}$:

$$\left.\begin{aligned} u/U_{\infty} &= u_{i0} + \varepsilon u_{i1} + \varepsilon^2 u_{i2} + \cdots \\ v/\varepsilon U_{\infty} &= v_{i0} + \varepsilon v_{i1} + \varepsilon^2 v_{i2} + \cdots \\ \rho/\rho_{\infty} &= \rho_{i0} + \varepsilon \rho_{i1} + \varepsilon^2 \rho_{i2} + \cdots \\ \mu/\mu_{\infty} &= \mu_{i0} + \varepsilon \mu_{i1} + \varepsilon^2 \mu_{i2} + \cdots \\ [p(X, Y) - p_w(X)]/\varepsilon \rho_{\infty} U_{\infty}^2 &= p_{i0} + \varepsilon p_{i1} + \varepsilon^2 p_{i2} + \cdots \end{aligned}\right\} \quad (2.1)$$

In this series, the $X = x/L$, $Y = y/\varepsilon L$, $\varepsilon = Re_{\infty}^{-1/2}$, $\frac{\partial}{\partial Y} = O(1)$; u_{i0}, u_{i1}, $\cdots$ quantities and their partial derivatives corresponding to X and Y are all of the O(1) quantitative level. We substitute expansion formula (2.1) into N-S equations (1.1) and (1.2), and by comparing the homogeneous powers of ε we obtain:

$$O(\varepsilon^0) \qquad \rho_{i0}\left(u_{i0}\frac{\partial u_{i0}}{\partial X} + v_{i0}\frac{\partial u_{i0}}{\partial Y}\right) = -\frac{d\bar{p}_w(X)}{dX} + \frac{\partial}{\partial Y}\left(\mu_{i0}\frac{\partial u_{i0}}{\partial Y}\right)$$

$$k_l \rho_{i0} u_{i0}^2 = \frac{\partial p_{i0}}{\partial Y} \tag{2.2}$$

$$O(\varepsilon^1) \qquad \rho_{i0}\left[\frac{\partial(u_{i0}u_{i1})}{\partial X} + v_{i0}\frac{\partial u_{i1}}{\partial Y} + v_{i1}\frac{\partial u_{i0}}{\partial Y} + k_l u_{i0} v_{i0}\right] + \rho_{i1}\left(u_{i0}\frac{\partial u_{i0}}{\partial X} + v_{i0}\frac{\partial u_{i0}}{\partial Y}\right)$$

$$= k_l Y\left(\rho_{i0}u_{i0}\frac{\partial u_{i0}}{\partial X} + \frac{d\bar{p}_w(X)}{dX}\right) - \frac{\partial p_{i0}}{\partial X} + \frac{\partial}{\partial Y}\left(\mu_{i0}\frac{\partial u_{i1}}{\partial Y} + \mu_{i1}\frac{\partial u_{i0}}{\partial Y}\right)$$

$$+ \varkappa\mu_{i0}\frac{\partial u_{i0}}{\partial Y} - k_l u_{i0}\frac{\partial \mu_{i0}}{\partial Y}$$

$$\rho_{i0}\left(u_{i0}\frac{\partial v_{i0}}{\partial X} + v_{i0}\frac{\partial v_{i0}}{\partial Y} - 2k_l u_{i0}u_{i1} + k_l Y u_{i0}^2\right) - \rho_{i1}k_l u_{i0}^2 = -\frac{\partial p_{i1}}{\partial Y}$$

$$+ \frac{4}{3}\frac{\partial}{\partial Y}\left(\mu_{i0}\frac{\partial v_{i0}}{\partial Y}\right) + \frac{\partial}{\partial X}\left(\mu_{i0}\frac{\partial u_{i0}}{\partial Y}\right) + j\frac{\sin\theta}{r_l}\mu_{i0}\frac{\partial u_{i0}}{\partial Y} \tag{2.3}$$

In the formulas, k_1=kL,, r_1=r/L, $\bar{p}_w(X) = p_w(X)/\rho_\infty U_\infty^2$; we can similarly carry out expansion of the continuity and energy equations. It is necessary to explain that in the present expansion method, when $y \leqslant \delta$, the O(1) quantitative levels of the N-S equations are close to being first order equation (2.2) and the O(ε) quantitative level equation is second order equation (2.3). They are all different from the common inner and outer layer matched expansion method [1]. In the common expansion method, when $y \leqslant \delta$, the first order and second order normal momentum equations are separately:

$$\frac{\partial p_{i0}}{\partial Y} = 0$$

$$\frac{\partial p_{i1}}{\partial Y} = k_l \rho_{i0} u_{i0}^2$$

Therefore, in the common expansion method, when using the first order equations of the inner and outer layers for simultaneous solution as well as when using second order equations for simultaneous solution, there always exists the mathematical

singularity of the $O\left(Re_\infty^{-1/2}\frac{U_\infty^2}{L}\right)$ quantitative level on the boundary line of y=δ.

In the y$>\delta$ non-viscous outer flow area, the solution of N-S equations (1.1) and (1.2) can be expanded to the following power series of ε

$$\left.\begin{aligned} u/U_\infty &= u_{e0} + \varepsilon u_{e1} + \varepsilon^2 u_{e2} + \cdots \\ v/U_\infty &= v_{e0} + \varepsilon v_{e1} + \varepsilon^2 v_{e2} + \cdots \\ \rho/\rho_\infty &= \rho_{e0} + \varepsilon \rho_{e1} + \varepsilon^2 \rho_{e2} + \cdots \\ p(X, Y_e)/\rho_\infty U_\infty^2 &= p_{e0} + \varepsilon p_{e1} + \varepsilon^2 p_{e2} + \cdots \end{aligned}\right\} \quad (2.4)$$

In the formulas, Y_e=y/L; the u_{eo},u_{el}... quantities and their partial derivatives corresponding to X and Y are all of the O(1) quantitative level. We substitute (2.4) into (1.1) and (1.2), compare the same order powers of ε and obtain:

$$O(\varepsilon^0) \quad \left.\begin{aligned} &\rho_{e0}\left(\frac{u_{e0}}{H}\frac{\partial u_{e0}}{\partial X} + v_{e0}\frac{\partial u_{e0}}{\partial Y_e} + \frac{k u_{e0} v_{e0}}{H}\right) = -\frac{1}{H}\frac{\partial p_{e0}}{\partial X} \\ &\rho_{e0}\left(\frac{u_{e0}}{H}\frac{\partial v_{e0}}{\partial X} + v_{e0}\frac{\partial v_{e0}}{\partial Y_e} - \frac{k u_{e0}^2}{H}\right) = -\frac{\partial p_{e0}}{\partial Y_e} \end{aligned}\right\} \quad (2.5)$$

$$O(\varepsilon^1) \quad \left.\begin{aligned} &\rho_{e0}\left[\frac{1}{H}\frac{\partial(u_{e0}u_{e1})}{\partial X} + v_{e0}\frac{\partial u_{e1}}{\partial Y_e} + v_{e1}\frac{\partial u_{e0}}{\partial Y_e} + \frac{k}{H}(u_{e1}v_{e0} + u_{e0}v_{e1})\right] \\ &\quad + \rho_{e1}\left(\frac{u_{e0}}{H}\frac{\partial u_{e0}}{\partial X} + v_{e0}\frac{\partial u_{e0}}{\partial Y_e} + \frac{k u_{e0} v_{e0}}{H}\right) = -\frac{1}{H}\frac{\partial p_{e1}}{\partial X} \\ &\rho_{e0}\left[\frac{u_{e0}}{H}\frac{\partial v_{e1}}{\partial X} + \frac{u_{e1}}{H}\frac{\partial v_{e0}}{\partial X} + \frac{\partial}{\partial Y_e}(v_{e0}v_{e1}) - \frac{2k u_{e0} u_{e1}}{H}\right] \\ &\quad + \rho_{e1}\left(\frac{u_{e0}}{H}\frac{\partial v_{e0}}{\partial X} + v_{e0}\frac{\partial v_{e0}}{\partial Y_e} - \frac{k u_{e0}^2}{H}\right) = -\frac{\partial p_{e1}}{\partial Y_e} \end{aligned}\right\} \quad (2.6)$$

We can see that in the non-viscous flow area of y $>\delta$, the O(1) quantitative levels of the N-S equations are close, that is, the first order equations in (2.5) are Euler equations. Accuracy reaches the approximation of the O(ε) quantitative level, that is, the second order equations in (2.6) which are the modified

solution after considering the boundary layer's displacement thickness effects. From the combined solution of equations (2.2) and (2.5), we can obtain a uniform solution in the entire flow field with the accuracy reaching the O(1) quantitative level; from the combined and matched solutions of equations (2.2), (2.3) and (2.5), (2.6), we can obtain the uniform solution in the entire flow field with the accuracy reaching the $O(Re_{\infty}^{1/2})$ quantitative level.

By comparing simplified N-S equations (1.4) and (1.5) with equations (2.2) and (2.3) as well as with (2.5) and (2.6), we can prove that the solution of the simplified N-S equations satisfies the following relationship and using u as the example we have

$$\left.\begin{array}{ll} y \leqslant \delta & \left|\dfrac{u}{U_{\infty}} - (u_{i0} + \varepsilon u_{i1})\right| \leqslant O(\varepsilon^2 - Re_{\infty}^{-1}) \\ y \geqslant \delta & \left|\dfrac{u}{U_{\infty}} - (u_{e0} + \varepsilon u_{e1})\right| \leqslant O(Re_{\infty}^{-1}) \end{array}\right\} \qquad (2.7)$$

As regards the other variables, formula (2.7) is established in the same way. Therefore, the solution of the simplified N-S equations is suitable for the uniform solution of the entire flow field with accuracy of the $O(Re_{\infty}^{1/2})$ quantitative level.

3. Concluding Remarks

The specific form of the simplified N-S equations will be slightly different because of the differences in the selection of the $O(Re_{\infty}^{1/2})$ quantitative level viscous terms. It is commonly considered that the actual difference caused by the different selection of $O(Re_{\infty}^{1/2})$ quantitative level viscous terms is not large [3-7]. However, in order to obtain a uniform solution in the entire flow field with accuracy of the $O(Re_{\infty}^{1/2})$ quantitative level, use of simplified N-S equations (1.3)-(1.6) proposed by the author [2] is possibly suitable.

The simplified N-S equations can correctly calculate the non-viscous flow, high entropic layer and complex perturbation between the boundary layer flows [3-7] in the shock wave layer. They are also widely used for the calculation of other flow fields: for example, nonequilibrium flows, duct flows, distant wake flows, shock wave-boundary layer perturbation as well as two dimensional pointed and dull viscous flows around bodies etc. The simplified N-S equations are also used for the calculation of complex flows such as separated flows, near wake flows and those with compressive corners etc. However, it should be noted that the simplified N-S equations only occupy a dominant position in viscous effects, that is, when $\mu \neq 0$, they possess parabolic-hyperbolic equation characteristics. When in $y > \delta$ viscous effects, they can be overlooked, that is, they transit to become Euler equations and the characteristic root is

$$\lambda_1 = \lambda_2 = H\frac{v}{u}, \quad \lambda_{3,4} = H\frac{uv \pm c\sqrt{M^2 - 1}}{u^2 - C^2}$$

Here, C is the speed of sound and M is the Mach number. When $M > 1$, $\lambda_{3,4}$ is the real character, it is hyperbolic and the mathematical type does not change; when $M < 1$, $\lambda_{3,4}$ is the repeated character, it is elliptic and the type of the simplified N-S equations changes. The Cauchy problem does not pertain to the simplified N-S equations of the elliptic type. It is essential to use iterative solutions or carry out other special processing.

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